

Final state radiation and a possibility to test a pion-photon interaction model near two-pion threshold.

G. Pancheri^{1)*}, O. Shekhovtsova^{1),2)†}, G. Venanzoni^{1)‡}

¹⁾*INFN Laboratori Nazionale di Frascati, Frascati (RM) 00044, Italy*

²⁾*NSC “Kharkov Institute for Physics and Technology”,
Institute for Theoretical Physics, Kharkov 61108, Ukraine*

Abstract

Final state radiation in the process $e^+e^- \rightarrow \pi^+\pi^-$ is considered for the cuts used in the analysis of KLOE data at large angles. By means of a Monte Carlo event generator FEVA, effects of non-pointlike behaviour of pions are estimated in the framework of Resonance Perturbation Theory. An additional complication related with the ϕ meson intermediate state is taken into account and the corresponding contributions (the direct decay $\phi \rightarrow \pi^+\pi^-\gamma$ and the double resonance decay $\phi \rightarrow \rho^\pm\pi^\mp \rightarrow \pi^+\pi^-\gamma$) are added to FEVA. A method to test effects of non-pointlike behaviour of pions in a model-independent way is proposed.

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1 Introduction

The ongoing experiments on precise measurements of the cross section of e^+e^- annihilation into hadrons aim to a precision at the 0.5 – 1.0% level [1, 2, 3]. Such an accuracy is crucial for various tests of the Standard Model [4], e.g., by confronting the experimentally measured value of a_μ [5], the muon anomalous magnetic moment, to the theoretical prediction. The accuracy of the theoretical calculation of a_μ is currently limited by the hadronic contribution, $a_\mu^{(had)}$. This contribution cannot be reliably calculated in the framework of perturbative QCD, because low-energy region dominates. Fortunately, its leading order part, $a_\mu^{(had;LO)}$, can be estimated from the dispersion relation using the experimental cross sections of e^+e^- annihilation as an input [4]

$$a_\mu^{(had;LO)} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{R(q^2)K(q^2)}{q^4} dq^2, \quad R(q^2) = \frac{\sigma_h(q^2)}{\sigma_\mu(q^2)}. \quad (1)$$

*e-mail: Giulia.Pancheri@lnf.infn.it

†e-mail: shekhovtsova@kipt.kharkov.ua

‡e-mail: Graziano.Venanzoni@lnf.infn.it

where $\sigma_h(q^2)$ is the total hadronic cross section, $\sigma_\mu(q^2)$ is the total cross section of the process $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$ and the function $K(q^2)$ is a smooth function that increases from 0.63 at the threshold ($s = 4m_\pi^2$) upto 1 for $q^2 \rightarrow \infty$. The quantity q^2 is the total four momentum squared of the final hadrons.

This behaviour of the integrand results in that the biggest contribution (about 70%) to the leading order hadronic part of the muon anomalous magnetic moment, $a_\mu^{(had;LO)}$, comes from the energy region below or about 1 GeV. Due to the presence of the ρ -meson the main contribution is related with the $\pi^+\pi^-$ final state.

Experimentally, the energy region from threshold to the collider beam energy is explored at the Φ -factory DAΦNE (Frascati, $s = 4E^2 = m_\phi^2$) [6, 7] and B-factory, PEP-2 (SLAC, $s = m_{Y(4S)}^2$) and KEKB (KEK, $s = m_{Y(4S)}^2$) [8, 9] using the method of radiative return [10, 11, 12]. This method relies on a factorization of the radiative cross section in the product of the hadronic cross section and a radiation function $H(q^2, \theta_{max}, \theta_{min})$ known from Quantum Electrodynamics (QED) [12, 13, 14]. The large luminosity of the Φ and B -factories allows to compensate the additional factor $\frac{\alpha}{2\pi}$ caused by the hard photon emission.

For a two-pion final state considered here it means that the radiative cross section $\sigma^{\pi\pi\gamma}$ corresponding to the process

$$e^+(p_1) + e^-(p_2) \rightarrow \pi^+(p_+) + \pi^-(p_-) + \gamma(k), \quad (2)$$

can be written as

$$q^2 \frac{d\sigma^{\pi\pi\gamma}}{dq^2} = \sigma^{\pi\pi}(q^2) H(q^2, \theta_{max}, \theta_{min}), \quad q = p_+ + p_-, \quad (3)$$

where the hadronic cross section $\sigma^{\pi\pi}$ is taken at a reduced CM energy. This factorization is valid only for photon radiation from the initial leptons (initial state radiation (ISR)). This is not possible for final state (FS) radiation (FSR) which is an irreducible background in radiative return measurements of the hadronic cross section [2, 15].¹ Indeed, the FSR cross section calculation has an additional complication compared to ISR case. In principle, RC caused by initial state radiation, i.e. the function $H(q^2, \theta_{max}, \theta_{min})$, can be calculated in QED, although the accuracy is technically limited. Instead, for the calculation of the FSR cross section the situation is different since its evaluation relies on models describing pion-photon interaction. Usually the combined sQED*VMD model is assumed as a model to calculate FS Bremsstrahlung process [12, 17]. In this case the pions are treated as point-like particles (the sQED model) and the total FSR amplitude is multiplied by the pion form factor, that is estimated in the VMD model. Unfortunately, the sQED*VMD model is an approximation that is valid for relatively soft photons and it can fail for high energy photons, i.e near the $\pi^+\pi^-$ threshold. In this energy region the contributions to FSR, beyond the sQED*VMD model, can become important. As it was shown in Ref. [18] the Resonance Perturbation Theory (RPT) is an appropriate model to describe photon-meson interactions in the energy region about 1 GeV and we will apply this model to estimate the Bremsstrahlung FS contributions beyond of sQED.

At the Φ factory DAΦNE there is an additional complication related with the possible intermediate ϕ meson state and the corresponding contributions should be included in the Monte Carlo event generator.

¹In fact the process of FSR cannot be excluded from the analysis. It can be suppressed choosing the small angle kinematics ($\theta_\gamma < \theta_{max} \ll 1$) but for the large angle analysis this contribution becomes upto 40% of ISR and should be estimated very carefully. (For some advantages of large angle analysis compared to small angle one see [16].)

In this paper we present the results obtained by the Monte Carlo event generator FEVA that simulates the process (2) for the DAΦNE accelerator setup. Our computer code FEVA was inspired by MC EVA [12]. The previous version of FEVA was described in [19], where the Bremsstrahlung process (both in the framework of RPT and sQED) and the ϕ direct decay (only f_0 parameterization) were considered. The current version of FEVA includes in addition the double resonance contribution and a more sophisticated parameterization for the ϕ direct decay.

This paper is organized as follows. In Section 2 we give a general description of FSR process and present the FSR models that have been already included in our program FEVA. In Section 3 the numerical results for the KLOE large angle analysis are presented. Since most of the effects arising in the FSR are model-dependent, we conclude this paper by suggesting a way to test possible effects beyond sQED, in a model-independent way (see Section 4). A conclusion is given in Section 5.

2 Final state radiation models

The cross section of the FSR process can be written as

$$d\sigma_F = \frac{1}{2s(2\pi)^5} \int \delta^4(Q - p_+ - p_- - k) \frac{d^3p_+ d^3p_- d^3k}{8E_+ E_- \omega} |M^{(FSR)}|^2, \quad (4)$$

where $Q = p_1 + p_2$, $s = Q^2$,

$$M^{(FSR)} = \frac{e}{s} M^{\mu\nu} \bar{u}(-p_1) \gamma_\mu u(p_2) \epsilon_\nu^*, \quad (5)$$

and where the FS tensor $M^{\mu\nu}$ describes the transition

$$\gamma^*(Q) \rightarrow \pi^+(p_+) \pi^-(p_-) \gamma(k).$$

It is convenient to parametrize the FS tensor in terms of three gauge invariant tensors (see [20] and Ref. [23, 24] in it):

$$\begin{aligned} M^{\mu\nu}(Q, k, l) &\equiv -ie^2 M_F^{\mu\nu}(Q, k, l) = -ie^2 (\tau_1^{\mu\nu} f_1 + \tau_2^{\mu\nu} f_2 + \tau_3^{\mu\nu} f_3), \\ \tau_1^{\mu\nu} &= k^\mu Q^\nu - g^{\mu\nu} k \cdot Q, \quad l = p_+ - p_-, \\ \tau_2^{\mu\nu} &= k \cdot l (l^\mu Q^\nu - g^{\mu\nu} k \cdot l) + l^\nu (k^\mu k \cdot l - l^\mu k \cdot Q), \\ \tau_3^{\mu\nu} &= Q^2 (g^{\mu\nu} k \cdot l - k^\mu l^\nu) + Q^\mu (l^\nu k \cdot Q - Q^\nu k \cdot l). \end{aligned} \quad (6)$$

We would like to emphasize that this expansion is totally model-independent. The model dependence is related only with an explicit form of the scalar functions f_i (we will call them structure functions).

Here is the list of the FSR processes included in FEVA MC:

$$e^+ + e^- \rightarrow \pi^+ + \pi^- + \gamma \quad \text{Bremsstrahlung process} \quad (7)$$

$$e^+ + e^- \rightarrow \phi \rightarrow (f_0; f_0 + \sigma) \gamma \rightarrow \pi^+ + \pi^- + \gamma \quad \phi \text{ direct decay} \quad (8)$$

$$e^+ + e^- \rightarrow \phi \rightarrow \rho^\pm \pi^\mp \rightarrow \pi^+ + \pi^- + \gamma \quad \text{VMD contribution} \quad (9)$$

$$e^+ + e^- \rightarrow \rho^\pm \pi^\mp \rightarrow \pi^+ + \pi^- + \gamma. \quad (10)$$

In the next sections we present the models describing these processes. The presence of (8) and (9) is due to the energy at which KLOE is running ($s = m_\phi^2$).

2.1 Bremsstrahlung process

As it was mentioned in Introduction, the sQED*VMD model is an approximation to describe soft photon radiation by pions. To estimate the contributions beyond the sQED*VMD model we applied RPT. The model is based on Chiral Perturbation Theory (χ PT) with the explicit inclusion of the vector and axial-vector mesons, $\rho_0(770)$ and $a_1(1260)$. Whereas χ PT gives correct predictions on the pion form factor at very low energy, RPT is the appropriate framework to describe the pion form factor at intermediate energies ($E \sim m_\rho$) [18]² and satisfies the QCD high energy behaviour.

Using the result of Ref. [20] we write the contribution to the functions f_i (see Eq.(6)) caused by the Bremsstrahlung FS process as

$$f_i = f_i^{sQED} + \Delta f_i^{RPT}, \quad (11)$$

$$f_1^{sQED} = \frac{2k \cdot Q F_\pi(Q^2)}{(k \cdot Q)^2 - (k \cdot l)^2}, \quad f_2^{sQED} = \frac{-2F_\pi(Q^2)}{(k \cdot Q)^2 - (k \cdot l)^2}, \quad (12)$$

$$f_3^{sQED} = 0, \quad (13)$$

where

$$\begin{aligned} \Delta f_1^{RPT} = & \frac{F_V^2 - 2F_V G_V}{f_\pi^2} \left(\frac{1}{m_\rho^2} + \frac{1}{m_\rho^2 - s - im_\rho \Gamma_\rho(s)} \right) \\ & - \frac{F_A^2}{f_\pi^2 m_a^2} \left[2 + \frac{(k \cdot l)^2}{D(l)D(-l)} + \frac{(s + k \cdot Q)[4m_a^2 - (s + l^2 + 2k \cdot Q)]}{8D(l)D(-l)} \right], \end{aligned} \quad (14)$$

$$\Delta f_2^{RPT} = -\frac{F_A^2}{f_\pi^2 m_a^2} \frac{4m_a^2 - (s + l^2 + 2k \cdot Q)}{8D(l)D(-l)}, \quad l = p_+ - p_-, \quad (15)$$

$$\Delta f_3^{RPT} = \frac{F_A^2}{f_\pi^2 m_a^2} \frac{k \cdot l}{2D(l)D(-l)}, \quad D(l) = m_a^2 - (s + l^2 + 2kQ + 4kl)/4. \quad (16)$$

For notations and details of the calculation we refer a reader to [20]. F_V , G_V and F_A are parameters of the model. According to the RPT model the pion form factor, that includes $\rho - \omega$ mixing, can be written as:

$$F_\pi(q^2) = 1 + \frac{F_V G_V}{f_\pi^2} B_\rho(q^2) \left(1 - \frac{\Pi_{\rho\omega}}{3q^2} B_\omega(q^2) \right), \quad (17)$$

where

$$B_r(q^2) = \frac{q^2}{m_r^2 - q^2 - im_r \Gamma_r(q^2)}, \quad (18)$$

q^2 is the virtuality of the photon, $f_\pi = 92.4$ MeV and the parameter $\Pi_{\rho\omega}$ describes $\rho - \omega$ mixing. An energy-dependent width is considered for the ρ meson:

$$\Gamma_\rho(q^2) = \Gamma_\rho \sqrt{\frac{m_\rho^2}{q^2}} \left(\frac{q^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2} \right)^{3/2} \cdot \Theta(q^2 - 4m_\pi^2), \quad (19)$$

²In that paper it was shown that the coupling constants of the effective chiral lagrangian at the order p^4 are essentially saturated by meson resonance exchange.

while for the ω -meson a constant width is used, $\Gamma_\omega = 8.68$ MeV, and $m_\omega = 782.7$ MeV. We assume that the parameter $\Pi_{\rho\omega}$, that determines ρ - ω mixing, is a constant and is related to the branching fraction $Br(\omega \rightarrow \pi^+\pi^-)$:

$$Br(\omega \rightarrow \pi^+\pi^-) = \frac{|\Pi_{\rho\omega}|^2}{\Gamma_\rho\Gamma_\omega m_\rho^2}. \quad (20)$$

The value of F_V and G_V as well as the mass of the ρ meson (m_ρ) and the parameter of the ρ - ω mixing $\Pi_{\rho\omega}$ were estimated by the fit of Novosibirsk CMD-2 data for the pion form factor [1]:

$$\begin{aligned} m_\rho &= 774.97 \pm 1.4 \text{ MeV}, \quad \Pi_{\rho\omega} = -2774 \pm 291.2 \text{ MeV}^2, \\ \Gamma_\rho &= 145.21 \pm 2.6 \text{ MeV}, \quad F_V = 154.22 \pm 0.5 \text{ MeV}. \end{aligned}$$

Then $G_V = 64.6 \pm 0.3$ MeV and $Br(\omega \rightarrow \pi^+\pi^-) = (0.96 \pm 0.19)\%$.

For the a_1 meson we take $m_a = 1.23$ GeV and $F_A = 0.122$ GeV corresponding to the mean value of the experimental decay width $\Gamma(a_1 \rightarrow \pi\gamma) = 640 \pm 246$ keV [21].

We would like to mention here that the contribution of any model describing the Bremsstrahlung FS process can be conveniently rewritten as in Eq.(11) and in the soft photon limit the results should coincide with the sQED*VMD model prediction.

2.2 ϕ direct decay

For DAΦNE energy ($s = m_\phi^2$) there are contributions to the final state $\pi^+\pi^-\gamma$ related with the intermediate ϕ meson state. In this section we consider the direct rare decay $\phi \rightarrow \pi^+\pi^-\gamma$.

The ϕ direct decay is assumed to proceed through the intermediate scalar meson state (either f_0 or $f_0 + \sigma$): $\phi \rightarrow (f_0; f_0 + \sigma)\gamma \rightarrow \pi^+\pi^-\gamma$, and its mechanism is described by a single form factor $f_\phi(Q^2)$. As it was shown in [22, 23], this process affects the form factor f_1 of Eq.(6):

$$f_1^{scal} = \frac{g_{\phi\gamma}f_\phi(Q^2)}{s - m_\phi^2 + im_\phi\Gamma_\phi}. \quad (21)$$

First, we consider the case of the f_0 intermediate state. To estimate this contribution we apply the Achasov four-quark model described in [24]: the $\phi \rightarrow f_0\gamma$ decay amplitude is generated dynamically through the loop of charged kaons. The form factor f_ϕ reads:

$$f_\phi^{K^+K^-}(Q^2) = \frac{g_{\phi K^+K^-}}{2\pi^2 m_K^2} \frac{g_{f_0\pi^+\pi^-} g_{f_0 K^+K^-} e^{i\delta_B(Q^2)}}{(m_{f_0}^2 - Q^2 + Re\Pi_{f_0}(m_{f_0}^2) - \Pi_{f_0}(Q^2))} I\left(\frac{m_\phi^2}{m_K^2}, \frac{Q^2}{m_K^2}\right), \quad (22)$$

where $I(.,.)$ is a function known in analytic form [22, 25] and $\delta_B(Q^2) = b\sqrt{Q^2 - 4m_\pi^2}$, $b = 75^\circ/\text{GeV}$. The term $Re\Pi_{f_0}(m_{f_0}^2) - \Pi_{f_0}(Q^2)$ takes into account the finite width corrections to the f_0 propagator [24]. A fit to the KLOE data $\phi \rightarrow \pi^0\pi^0\gamma$ ³ gives the following values of the parameters [26]:

$$m_{f_0} = 0.962 \text{ GeV}, \quad \frac{g_{f_0 K^+K^-}^2}{4\pi} = 1.29 \text{ GeV}^2, \quad \frac{g_{f_0 K^+K^-}^2}{g_{f_0\pi^+\pi^-}^2} = 3.22. \quad (23)$$

³ $\Gamma(f_0 \rightarrow \pi^+\pi^-) = \frac{2}{3}\Gamma(f_0 \rightarrow \pi\pi)$

In a refined version of this model which includes the σ meson in the intermediate state [27], the form factor f_ϕ can be written as

$$f_\phi^{K^+K^-}(Q^2) = \frac{g_{\phi K^+K^-}}{2\pi^2 m_K^2} e^{i(\delta_{\pi\pi}(Q^2) + \delta_{KK}(Q^2))} I\left(\frac{m_\phi^2}{m_K^2}, \frac{Q^2}{m_K^2}\right) \sum_{R,R'} g_{RK^+K^-} G_{RR'}^{-1} g_{R'\pi^+\pi^-},$$

where $G_{RR'}$ is the matrix of inverse propagators [27]. Such an extension of the model improves the description of the data at low Q^2 (see Fig. 1, *left*) and gives the following value of the model parameters [28]

$$m_{f_0} = 0.977 \text{ GeV}, \quad \frac{g_{f_0 K^+K^-}^2}{4\pi} = 1.12 \text{ GeV}^2, \quad \frac{g_{f_0 K^+K^-}^2}{g_{f_0 \pi^+\pi^-}^2} = 6.9, \quad (24)$$

$$m_\sigma = 0.462 \text{ GeV}, \quad \frac{g_{\sigma K^+K^-}^2}{4\pi} = 0.024 \text{ GeV}^2, \quad \frac{g_{\sigma K^+K^-}^2}{g_{\sigma \pi^+\pi^-}^2} = 0.052. \quad (25)$$

2.3 VMD contribution

Another contribution producing the intermediate ϕ meson state is the double resonance contribution (9). In this case the off-shell ϕ meson decays to ρ^\pm and π^\mp followed by $\rho \rightarrow \pi\gamma$. The explicit value for the functions f_i^{VMD} for this decay can be found in Ref. [23]. To correspond to the KLOE analysis [28] we added also the additional phase between VMD and ϕ direct contributions, the factor Π_ρ^{VMD} ⁴ and the phase of the ω - ϕ meson mixing $\beta_{\omega\phi}$:

$$\begin{aligned} f_1^{VMD} &= -\frac{1}{4\pi\alpha s} \left((-1 + \frac{3}{2}x + \sigma)(g(x_1) + g(x_2)) \right. \\ &\quad \left. + \frac{1}{4}(x_1 - x_2)(g(x_1) - g(x_2)) \right), \\ f_2^{VMD} &= -\frac{1}{4\pi\alpha s^2} (g(x_1) + g(x_2)), \\ f_3^{VMD} &= -\frac{1}{8\pi\alpha s^2} (g(x_1) - g(x_2)), \end{aligned} \quad (26)$$

where

$$g(x) = \frac{e g_{\rho\pi}^\phi g_{\pi\gamma}^\rho}{4F_\phi} \frac{m_\phi^2 e^{i\beta_\rho} e^{i\beta_{\omega\phi}}}{s - m_\phi^2 + im_\phi\Gamma_\phi} \frac{s^2 \Pi_\rho^{VMD}}{(1-x)s - m_\rho^2 + im_\rho\Gamma_\rho((1-x)s)} \quad (27)$$

with $x_{1,2} = \frac{2p_{+,-}(p_1 + p_2)}{s}$ and $x = 2 - x_1 - x_2$. The quantities $g_{\rho\pi}^\phi$, $g_{\pi\gamma}^\rho$ are the coupling constants determining the $\phi \rightarrow \rho\pi$ and $\rho \rightarrow \pi\gamma$ vertexes, respectively, $F_\phi = \sqrt{\frac{3\Gamma(\phi \rightarrow e^+e^-)}{\alpha m_\phi}}$ and $e = \sqrt{4\pi\alpha}$. A fit to the KLOE data $\phi \rightarrow \pi^0\pi^0\gamma$ [28] gives:

$$\begin{aligned} g_{\rho\pi}^\phi &= 0.811 \text{ GeV}^{-1}, \quad g_{\pi\gamma}^\rho = 0.295 \text{ GeV}^{-1}, \quad F_\phi = 42.5, \quad \Pi_\rho^{VMD} = 0.58195, \\ \beta_\rho &= 32.996^\circ, \quad \beta_{\omega\phi} = 163^\circ. \end{aligned} \quad (28)$$

⁴Including Π_ρ^{VMD} one rescales the coupling constant. In our opinion it rescales the constant $g_{\rho\pi}^\phi$ that cannot be directly determined from any experimental decay width.

2.4 Other contributions.

We included in our program the channel $\gamma^* \rightarrow \rho^\pm \pi^\mp \rightarrow \pi^+ \pi^- \gamma$ ⁵, whose amplitude has been evaluated in RPT model. To write this part of FSR we used the results of Ref. [20] for the function $f_i^{\rho^\pm}$:

$$\Delta f_1^{\rho^\pm} = \frac{8H_V^2}{9f_\pi^2} \left[(k \cdot Q + l^2) \left(\frac{1}{C(l)} + \frac{1}{C(-l)} \right) + 2k \cdot l \left(\frac{1}{C(l)} - \frac{1}{C(-l)} \right) \right] + \frac{64H_V^2}{9f_\pi^2}, \quad (29)$$

$$\Delta f_2^{\rho^\pm} = -\frac{8H_V^2}{9f_\pi^2} \left(\frac{1}{C(l)} + \frac{1}{C(-l)} \right), \quad (30)$$

$$\Delta f_3^{\rho^\pm} = \frac{8H_V^2}{9f_\pi^2} \left(\frac{1}{C(l)} - \frac{1}{C(-l)} \right), \quad (31)$$

where $C(\pm l) = m_\rho^2 - (k + p_\pm)^2 - im_\rho \Gamma_\rho ((k + p_\pm)^2)$ with $(k + p_\pm)^2 = (Q^2 + l^2 + 2k \cdot Q \pm 4k \cdot l)/4$. A value of the constant H_V is determined by the width of the $\rho \rightarrow \pi\gamma$ decay

$$\Gamma(\rho^\pm \rightarrow \pi^\pm \gamma) = \frac{4\alpha m_\rho^3 H_V^2}{27f_\pi^2} \left(1 - \frac{m_\pi^2}{m_\rho^2} \right)^3$$

and can be related with the constant $g_{\pi\gamma}^\rho$: $H_V = \frac{3f_\pi g_{\pi\gamma}^\rho}{4\sqrt{2}}$. It gives $H_V = 0.0144$. In agreement with the calculation given in [20], we found a negligible contribution of this channel and, for the sake of simplicity, we discard its effects on the numerical results presented in the following section.

3 Numerical results

In this chapter we present the results for the differential cross section and the forward-backward asymmetry [12, 29] for the reaction $e^+e^- \rightarrow \pi^+ \pi^- \gamma$, where the FSR amplitude (M_{FSR}) receives contributions from both RPT (M_{RPT}) and the $\phi \rightarrow \pi^+ \pi^- \gamma$ decay (M_ϕ). The last one is a sum of the ϕ direct decay (M_ϕ^{scal}) and VMD (M_ϕ^{VMD}) contributions. Thus the total contribution of the process (2) $d\sigma_T$ can be written as:

$$\begin{aligned} d\sigma_T &= d\sigma_I + d\sigma_F + d\sigma_{IF} \sim |M_{ISR} + M_{FSR}|^2, \\ d\sigma_I &\sim |M_{ISR}|^2, \\ d\sigma_F &\sim |M_{RPT}|^2 + |M_\phi|^2 + 2\text{Re}\{M_{RPT} \cdot M_\phi^*\}, \\ d\sigma_{IF} &\sim 2\text{Re}\{M_{ISR} \cdot (M_{RPT} + M_\phi)^*\} \end{aligned} \quad (32)$$

where $d\sigma_I$ corresponds to the ISR cross section, $d\sigma_F$ is for the FSR one. The interference term $d\sigma_{IF}$ is equal to zero for symmetric cuts on the polar angle of the pions [7].

The different contributions to the FSR differential cross section $d\sigma_F$, evaluated at $s = m_\phi^2$, are shown in Fig.1, *right*, for the full angular range $0^\circ \leq \theta_\gamma \leq 180^\circ, 0^\circ \leq \theta_\pi \leq 180^\circ$. Good agreement between the results of the Monte Carlo simulation (points), with the analytic

⁵In the energy region $s \leq m_\phi^2$ this direct transition $\gamma^* \rightarrow \rho^\pm \pi^\mp$ can be considered as the tail of the double resonance contribution of the ρ' meson decay: $\gamma^* \rightarrow \rho' \rightarrow \rho\pi$ for $s = m_\phi^2$

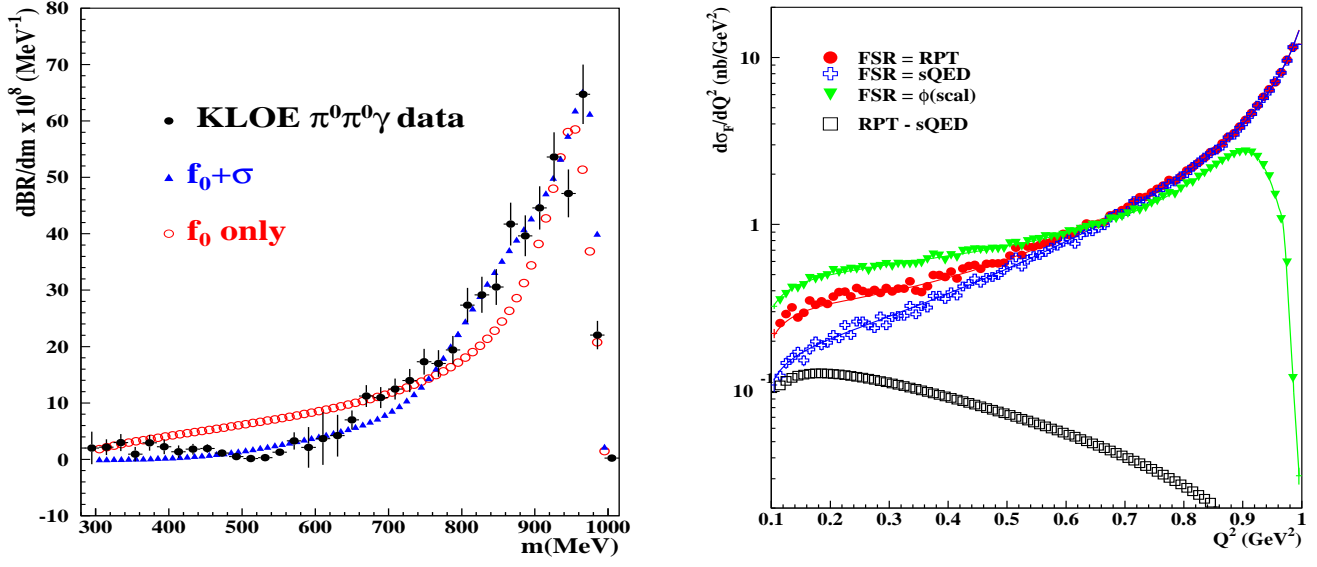


Figure 1: (*Left*) The dependence of the branching ratio of the ϕ direct decay on the intermediate scalar states. (*Right*) Contribution to the FSR cross section $d\sigma_F/dQ^2$ in the region $0^\circ \leq \theta_\gamma \leq 180^\circ$, $0^\circ \leq \theta_\pi \leq 180^\circ$ at $s = m_\phi^2$. RPT is represented by circles, sQED by crosses, ϕ by triangles, while the difference between RPT and sQED is indicated by squares.

prediction (solid line) is found. It can be noted that at low Q^2 the contribution from the direct ϕ decay (i.e. the term proportional to $|M_\phi^{scal}|^2$ in Eq. (32)) is quite large and, therefore, the additional contribution beyond sQED, can be revealed only in the case of destructive interference between the two amplitudes ($\text{Re}(M_{RPT} \cdot M_\phi^*) < 0$). Published data from the KLOE experiment [2] are in favour of this assumption, which we will use in the following.

To begin with, we consider the case $s = m_\phi^2$. In Fig.2 we show the values of the differential cross section $d\sigma_T/d\sigma_I$ and the forward-backward asymmetry for the angular cuts of the KLOE large angle analysis [16, 30]:

$$\begin{aligned} 50^\circ &\leq \theta_\gamma \leq 130^\circ, \\ 50^\circ &\leq \theta_\pi \leq 130^\circ \end{aligned} \quad (33)$$

for the Bremsstrahlung FS process in the framework of the sQED*VMD model and with the ϕ decay contributions (VMD and the ϕ direct decay), for a hard photon radiation with energies $E_\gamma > 20$ MeV.

Figure 3 shows the effects of RPT and ϕ direct decay terms to the total differential cross section and their contribution to FSR one for angular cuts (33). Several distinctive features can be noted: (1) the peak at about 1 GeV^2 corresponds to the f_0 intermediate state for the direct $\phi \rightarrow \pi\pi\gamma$ amplitude; (2) the presence of RPT terms in the FSR is relevant at low Q^2 , where they give an additional contribution up to 40% to the ratio $d\sigma_{RPT+\phi}/d\sigma_{sQED+\phi}$; (as shown in Fig. 3, *left*); (3) the destructive interference between the ϕ decay and the Bremsstrahlung FS process decreases the visible cross section in the whole Q^2 region and its dependence on FS Bremsstrahlung model at low Q^2 (see Fig. 2, *left*, down). Also we would like to draw attention to the VMD contribution. As we can see from Fig. 2, the VMD contribution almost does not change the value of the differential cross section (Fig. 2, *left*), but it changes essentially the

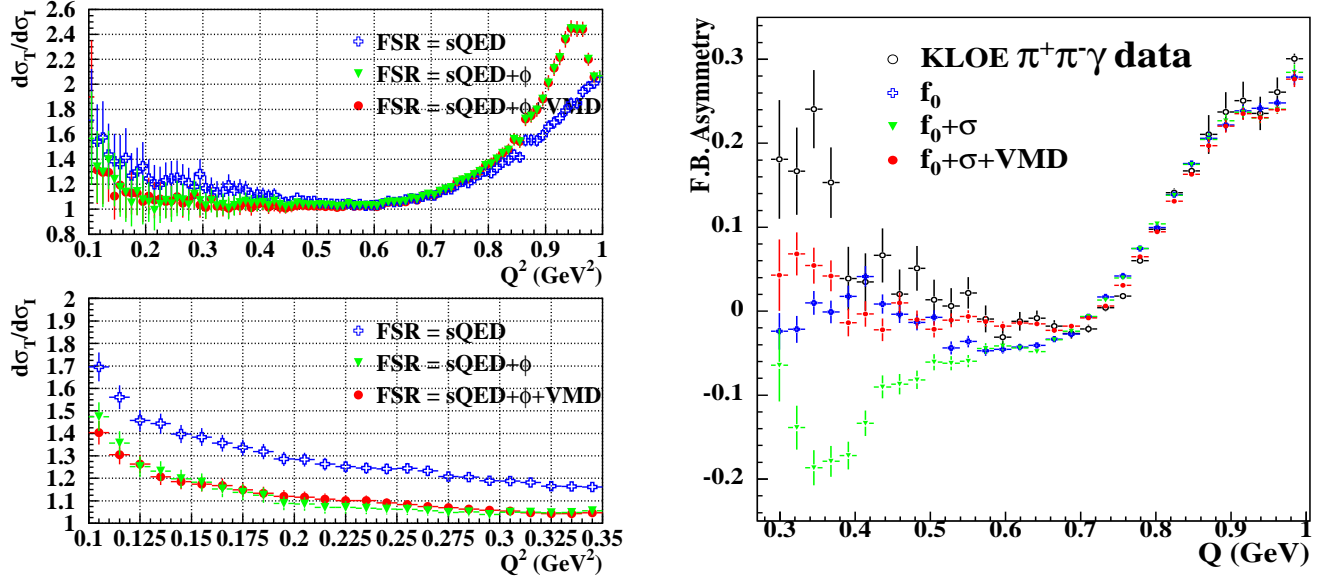


Figure 2: The ratio of the total cross section with respect to ISR one $d\sigma_T/d\sigma_I$ (*left*) and the forward-backward asymmetry (*right*) as function of the invariant mass of the two pions, when the ϕ contribution is taken into account and the Bremsstrahlung process is in the framework of the sQED*VMD model. The angular region is $50^\circ \leq \theta_\gamma \leq 130^\circ$, $50^\circ \leq \theta_\pi \leq 130^\circ$ and $s = m_\phi^2$.

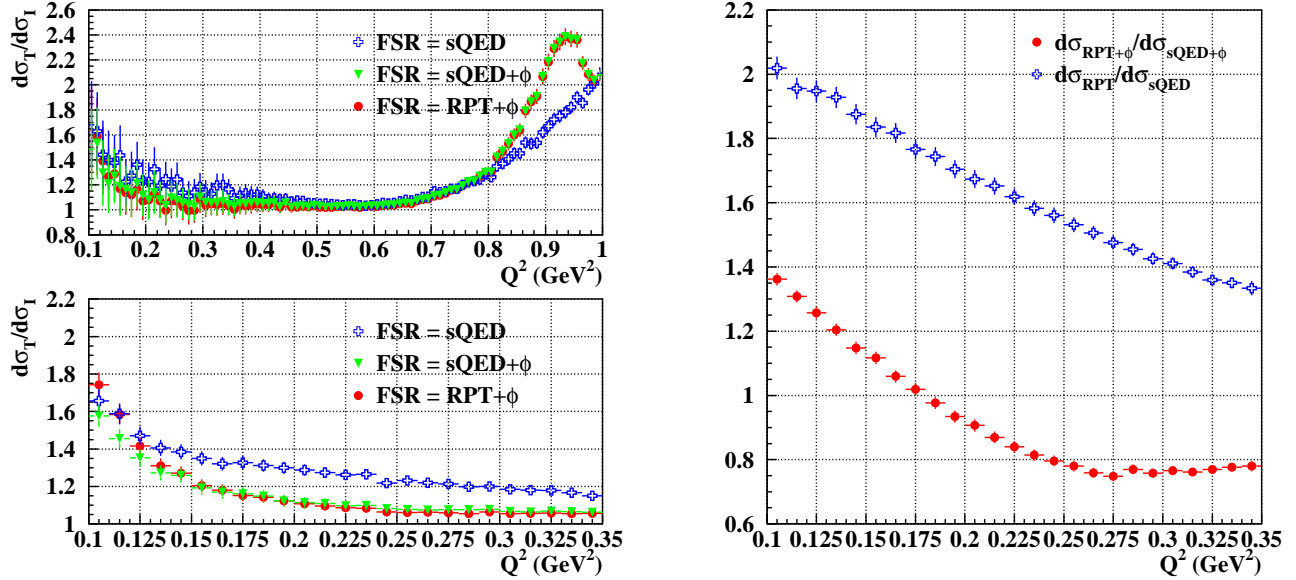


Figure 3: The ratio $d\sigma_T/d\sigma_I$ (*left*) as function of the invariant mass of the two pions for different models describing the Bremsstrahlung FS process (either RPT or sQED*VMD) and the ratio of FSR cross section in the framework of RPT, with respect to sQED, when the ϕ direct decay contribution is (or not) taken into account (*right*), in the region $50^\circ \leq \theta_\gamma \leq 130^\circ$, $50^\circ \leq \theta_\pi \leq 130^\circ$ at $s = m_\phi^2$.

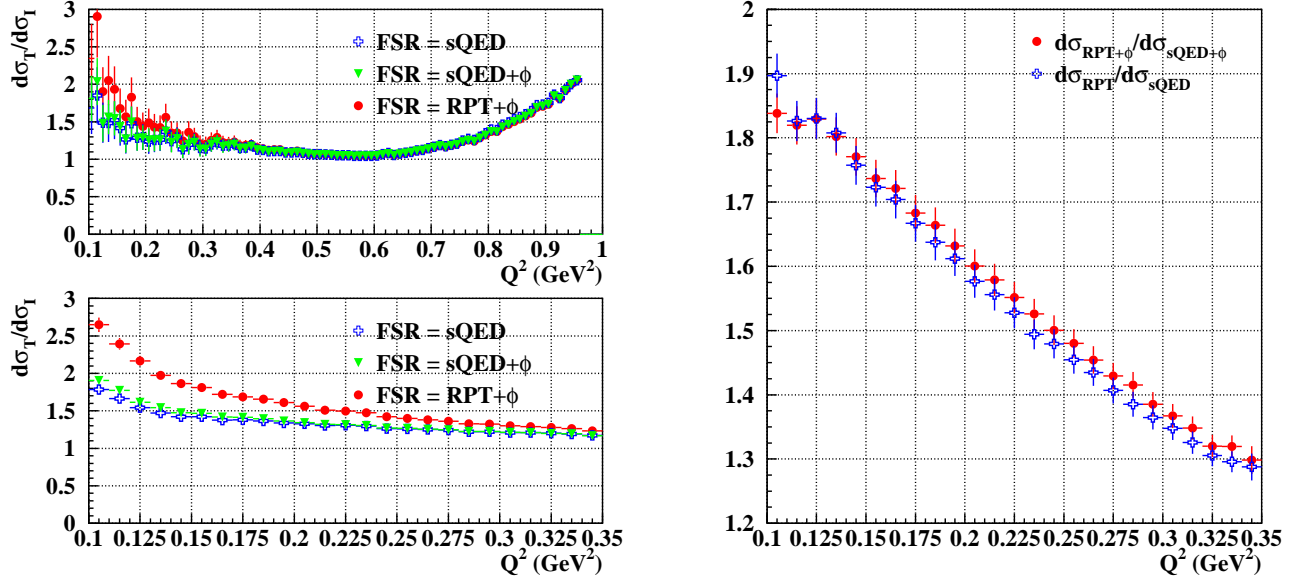


Figure 4: The same ratio as in Fig. 3 at $s = 1\text{GeV}^2$.

value of the forward-backward asymmetry (Fig. 2, *right*) and it follows the experimental data for it. Last but not least, all contributions beyond sQED are large enough near the threshold to make the analysis difficult.

In order to reduce the background from ϕ decay in the measurement of the pion form factor at threshold, KLOE has taken more than 200 pb^{-1} of data at 1 GeV [31]. In this case the ϕ meson intermediate contributions are suppressed (see Fig. 4, (right), the values of $d\sigma_T$ with and without the ϕ decay almost coincide) and the main contribution to FSR comes from Bremsstrahlung process (see Section 2.1), allowing to study a model for it.

4 Model-independent test of FSR models

Contributions to Bremsstrahlung FS process beyond sQED, as in the case of RPT, can lead to sizeable effects on the cross section and asymmetry at threshold, as shown in Figs. 2-4. Precise measurement of the pion form factor in this region needs to control them at the required level of accuracy. This looks like a rather difficult task, if one thinks that effects beyond sQED, as well as the contribution from $\phi \rightarrow \pi^+\pi^-\gamma$, are model-dependent.

One can think to construct a general amplitude for the $e^+e^- \rightarrow \pi^+\pi^-\gamma$, according to some underlying theory, and try to determine the free parameters by a constrained fit on specific variables (like mass spectrum, charge and forward-backward asymmetry, angular distribution, etc...). Particularly for the charge asymmetry, it has been proved to be a powerful tool to discriminate between different models of $\phi \rightarrow \pi^+\pi^-\gamma$ [29]. However, when the number of the parameters is large, correlations between the parameters of the model can arise and spoil the effective power of these fits. The situation becomes even worse if the pion form factor also has to be extracted from the same data. As an example, in the case of RPT model, if we consider only the ρ and ω contribution to the pion form factor and the ρ and a_1 contribution to FSR the number of free parameters is already six. The presence of the ϕ direct and VMD decays results in additional free parameters.

The possibility to determine some of the parameters by external data can strongly help, as in the case of the $\phi \rightarrow \pi^+\pi^-\gamma$ amplitude, which can be determined by the $\pi^0\pi^0\gamma$ channel copiously produced at DAΦNE. An additional source of information, which will be used to determine the contributions to FSR beyond sQED in a model-dependent way, is the dependence of the FSR amplitude on the e^+e^- invariant mass squared s .

Let us write the differential cross section for the emission of one photon in the process $e^+e^- \rightarrow \pi^+\pi^-\gamma$ as a function of the invariant mass of the two pions:

$$\left(\frac{d\sigma_T}{dQ^2}\right)_s = |F_\pi(Q^2)|^2 H_s(Q^2) + \left(\frac{d\sigma_F}{dQ^2}\right)_s, \quad (34)$$

where $\left(\frac{d\sigma_F}{dQ^2}\right)_s$ is the differential cross section for the emission of a photon in the final state, while the ISR function $H_s(Q^2)$ was defined in the Introduction. We indicate by a subscript s the dependence of each quantity on the e^+e^- invariant mass (s). Since we will consider only symmetric angular cuts for pions, the interference term between initial and final state radiation has been neglected.

At relatively high Q^2 the FSR differential cross section, $\left(\frac{d\sigma_F}{dQ^2}\right)_s$, is dominated by the contribution coming from sQED (M_{sQED}) and ϕ direct decay (M_ϕ):

$$\left(\frac{d\sigma_{sQED+\phi}}{dQ^2}\right)_s \sim |M_{sQED} + M_\phi|^2. \quad (35)$$

Contributions beyond sQED (ΔM) are expected to be important at low Q^2 . They introduce an additional term (ΔM) in the above expression:

$$\left(\frac{d\sigma_F}{dQ^2}\right)_s \sim |M_{sQED} + \Delta M + M_\phi|^2 = \quad (36)$$

$$= |M_{sQED} + M_\phi|^2 + |\Delta M|^2 + 2\text{Re}\left\{\Delta M \cdot (M_{sQED} + M_\phi)^*\right\}. \quad (37)$$

We will now consider the following quantity:

$$Y_s(Q^2) = \frac{\left(\frac{d\sigma_T}{dQ^2}\right)_s - \left(\frac{d\sigma_{sQED+\phi}}{dQ^2}\right)_s}{H_s(Q^2)} = |F_\pi(Q^2)|^2 + \Delta F_s(Q^2), \quad (38)$$

where $\Delta F_s \sim \left(|\Delta M|^2 + 2\text{Re}\left\{\Delta M \cdot (M_{sQED} + M_\phi)^*\right\}\right)_s / H_s$.

If no contribution beyond sQED is present ($\Delta M = 0$), $Y_s(Q^2)$ coincides with the square of the pion form factor, **independently of the energy \sqrt{s} at which it is evaluated**, while any dependence on s is only due to an additional contribution to FSR beyond sQED. In particular, the difference of $Y_s(Q^2)$ computed at two beam energies (s_1 and s_2), can only come from FSR beyond sQED:

$$\Delta Y(Q^2) = Y_{s_1}(Q^2) - Y_{s_2}(Q^2) = \Delta F_{s_1}(Q^2) - \Delta F_{s_2}(Q^2) \quad (39)$$

Therefore, before extracting the pion form factor at threshold, we suggest to look at the difference $\Delta Y(Q^2)$, which can be used to estimate the contribution beyond sQED to the FSR amplitude in a model-independent way.

As a realistic application of this procedure, we consider the case of DAΦNE, where KLOE has already collected more than 200 pb⁻¹ at 1 GeV² and 2.5 fb⁻¹ at m_ϕ^2 , that, in the range

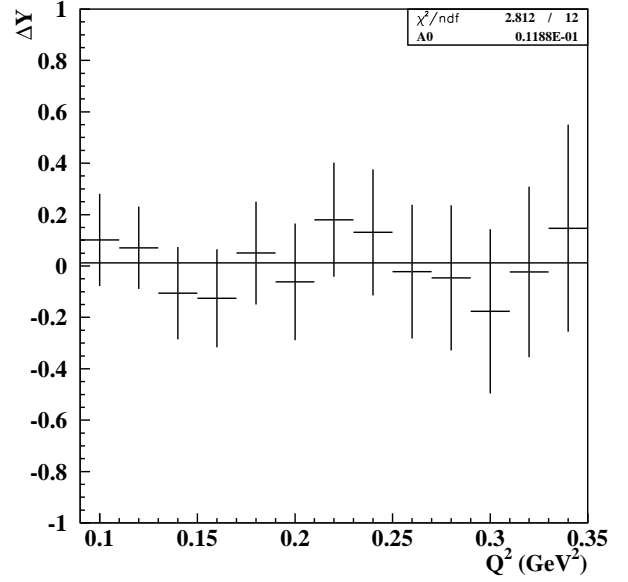
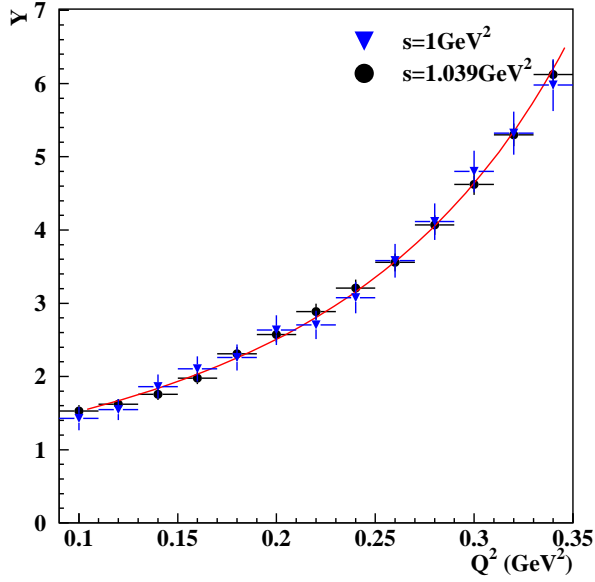


Figure 5: *Left:* $Y_s(Q^2)$ at $s = 1 \text{ GeV}^2$ (triangles), and at $s = m_\phi^2$ (circles), when FSR includes only the sQED and ϕ contribution. The pion form factor $|F_\pi(Q^2)|^2$ is shown by a solid line. *Right:* The difference $\Delta Y(Q^2)$.

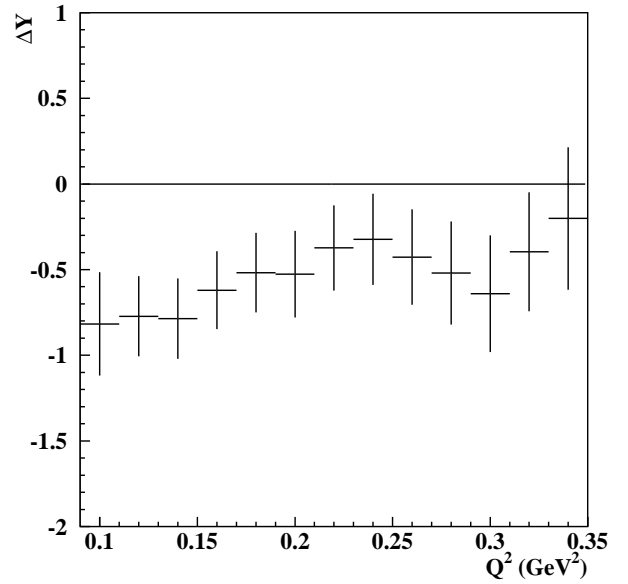
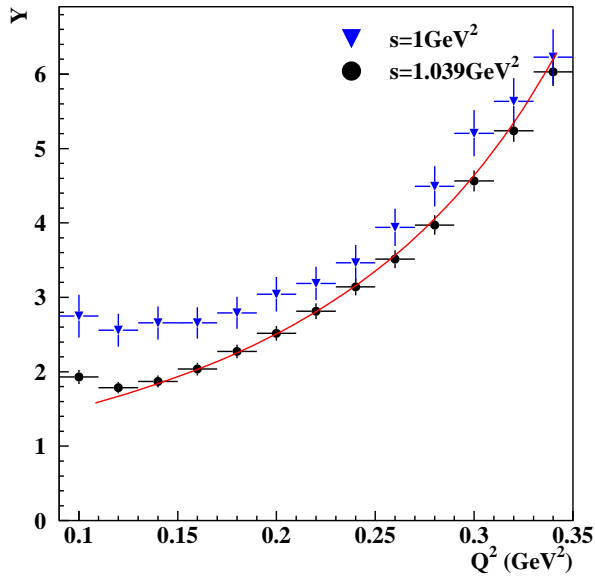


Figure 6: *Left:* $Y_s(Q^2)$ at $s = 1 \text{ GeV}^2$ (triangles), and at $s = m_\phi^2$ (circles), when FSR includes RPT and ϕ contribution. The pion form factor $|F_\pi(Q^2)|^2$ is shown by a solid line. *Right:* The difference $\Delta Y(Q^2)$.

$Q^2 < 0.35 \text{ GeV}^2$, correspond to $O(10^3)$ and $O(10^4)$ events, respectively, in the region $50^\circ \leq \theta_\gamma \leq 130^\circ$, $50^\circ \leq \theta_\pi \leq 130^\circ$. We will consider RPT as a model for the effects beyond sQED.

Fig. 5, *left*, shows the quantity $Y_s(Q^2)$ at $s_1 = 1 \text{ GeV}^2$ and at $s_2 = m_\phi^2$ when no additional RPT term is included in FSR. As expected, each of these quantities coincides with the square of the pion form factor $|F_\pi(Q^2)|^2$, shown by a solid line. The difference $\Delta Y(Q^2)$ is shown in Fig.5, *right*, which is consistent with zero as expected. In the region $s < 0.35 \text{ GeV}^2$ one can expand the pion form factor as it was done in [4]:

$$F_\pi(q^2) \simeq 1 + p_1 \cdot q^2 + p_2 \cdot q^4. \quad (40)$$

Using the same experimental data for the pion form factor [1] as before we have: $p_1 = 1.15 \pm 0.06 \text{ GeV}^{-2}$, $p_2 = 9.06 \pm 0.25 \text{ GeV}^{-4}$, $\chi^2/\nu \simeq 0.13$. A combined fit of $Y_s(Q^2)$ to the pion form factor gives the following values: $p_1 = 1.4 \pm 0.186 \text{ GeV}^{-2}$, $p_2 = 8.8 \pm 0.73 \text{ GeV}^{-4}$, $\chi^2/\nu = 0.25$, that is in a reasonable agreement with the results (40).

The situation is different as soon as the Bremsstrahlung FS process is modeled by RPT. In this case, as shown in Fig.6, *right*, the difference $\Delta Y(Q^2) \neq 0$ and the quantities $Y_s(Q^2)$ cannot be anymore identified with $|F_\pi(Q^2)|^2$, (see Fig.6, *left*)⁶.

Before concluding, we would like to point out the main points of our present method:

- The quantity $\left(\frac{d\sigma_{sQED+\phi}}{dQ^2}\right)_s$ is an input parameter of our procedure, and can be computed numerically by Monte Carlo;
- The amplitude for $\phi \rightarrow \pi^+\pi^-\gamma$ is taken from the $\pi^0\pi^0\gamma$ channel.
- missing ISR multi-photon radiative correction can be added in H_s and it will not spoil the effective power of the method;
- A clear advantage of the procedure based on a Monte Carlo event generator is that it allows to keep control over efficiency and resolution of the detector and fine tuning of the parameters.

Even if the main limitation of the method could come from the uncertainty on the parameters of $\phi \rightarrow \pi^+\pi^-\gamma$ amplitude, especially at low Q^2 , we believe that the KLOE data on $\phi \rightarrow \pi^0\pi^0\gamma$ will allow a precise description of this amplitude. In any case, in agreement with [29] we strongly recommend to check the amplitude by using charge asymmetry and to compare with spectrum of the $\pi^+\pi^-\gamma$, at least at high Q^2 , where the pointlike approximation is safe (as done in [30]).

5 Conclusion

A test of FSR at threshold in the process $e^+e^- \rightarrow \pi^+\pi^-\gamma$ is a rather important issue, not only for the role of FSR as background to the measurement of the pion form factor, but also to get information about pion-photon interactions when the intermediate hadrons are far off shell. At $s = m_\phi^2$ an additional complication arises: the presence of the decay $\phi \rightarrow \pi^+\pi^-\gamma$, that goes either through the intermediate scalar (the direct ϕ decay) or the vector state (VMD contribution) whose amplitude and relative phase can be described according to some model.

⁶Destructive interference between RPT and $\phi \rightarrow \pi^+\pi^-\gamma$ amplitudes tends to cancel out the effects beyond sQED at $s = m_\phi^2$ (see Fig. 3, *left*). Therefore the quantity $Y_s(Q^2)$ almost coincides with the pion form factor.

By means of a Monte Carlo event generator FEVA, which also includes the contribution of the rare $\phi \rightarrow \pi^+\pi^-\gamma$ decay, we estimate the effects beyond sQED in the framework of Resonance Perturbation Theory (RPT) for angular cuts used in the KLOE analysis of the pion form factor at threshold. We show that the low Q^2 region is sensitive both to the inclusion of additional terms in the FSR amplitude given by the RPT model and to the ϕ decay contribution (especially its VMD part).

We also propose a method which allows to estimate the effects beyond sQED in a model-independent way. We found that the deviation from sQED predicted by RPT can be observed within the current KLOE statistics.

We would like to emphasize once again that this work was motivated by ongoing experiment on precise measurements of the muon anomalous magnetic moment [32] that allows to perform tests of the Standard Model with a fabulous precision.

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